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TECHNICAL MEMORANDUMS  
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No. 488

DETERMINATION OF THE AIR FORCES AND MOMENTS  
PRODUCED BY THE AILERONS OF AN AIRPLANE

By C. Wieselsberger and T. Asano

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E R R A T A

Page 11, line 7, symbols should read:

$$W_1 = q b^2 \eta \alpha_q^2$$

Page 11, line 8, symbols should read:

$$M_Z = q b^3 \xi \alpha_q \alpha$$

Page 11, line 11, symbols should read:

$$W_2 = q b^2 \kappa \alpha_q^2$$

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DETERMINATION OF THE AIR FORCES AND MOMENTS

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I.

The quantitative knowledge of the air forces and moments produced by the deflection of the ailerons of an airplane is of considerable importance to the airplane builder for maneuverability investigations, as well as for the strength calculation of the wings. In a work previously published by one of the authors, full particulars relative to the method employed in computing these forces and moments were given, and the results of numerical calculations obtained under certain assumptions were also presented.\*\* On account of the great number of variables affecting this problem, the results therein obtained are not applicable in all cases. On the contrary, they are valid only for a definite aspect ratio and a definite slope of the lift curve against the angle of attack; or, speaking more precisely, they are valid only for a certain nondimensional parameter  $p$  obtained by division of both these quantities.

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\*"Bestimmung der durch die Querruder eines Tragflügels erzeugten Luftkräfte und Moments," Zeitschrift für Flugtechnik und Motorluftschiffahrt, July 14, 1928.

\*\*C. Wieselsberger, "Theoretische Untersuchungen über die Querruderwirkung beim Tragflügel." Report of the Aeronautical Research Institute, Tokyo Imperial University, No. 30, 1927.

The exact determination of these aerodynamic quantities for other values of the parameter  $p$  would require further quite extensive numerical computations, similar to those made in the above-mentioned work. It would therefore be desirable to derive approximate results for other values of the parameter from the above-mentioned results. A rough approximation method for another law of lift has already been given in a note in the above-mentioned work. Subsequently it was shown that a considerably better method can be given, by means of which the magnitude of the air forces and moments may be determined with sufficient precision from the present results for all values of  $p$  occurring in practice. There is no object in carrying the approximation too far, since, as a rule, the actual lift distribution at the wing tips shows deviations from the theoretical distribution, so that an approximate evaluation of the distribution may be regarded as satisfactory. After a few brief remarks on the fundamentals of the exact computation, the method will be so presented that the lift distribution for deflected ailerons may be determined for other values of the parameter  $p$  from the results already obtained. Coefficients will then be given in the form of diagrams and numerical tables, from which the desired forces and moments can be easily obtained by substitution in the given equations.

## II.

In regard to the details of the exact computation we must refer to the above-mentioned work. The method employed was essentially derived from E. Trefftz.\* It is also found in H. Glauert's book\*\* in a somewhat different form. Here we can only mention that the lift distribution is expressed by a Fourier series, the coefficients of which are so computed from a system of linear equations as to satisfy the conditions of Prandtl's wing theory lying at the basis of the whole computation. The fundamental equation, which expresses the requirements of the wing theory, must, strictly speaking, be satisfied at all points along the wing span. In the present case we have confined ourselves to 16 points. Due to certain characteristics of symmetry, it is only necessary to carry out the calculation for one wing. The calculation then consists in the solution of eight linear equations with eight unknown quantities. In this connection we must also notice that the discontinuous change of the angle of attack, as is actually the case with aileron deflections, is represented by a transition region of finite length. Therefore we do not obtain an exact solution, but one which gives, nevertheless, sufficiently accurate results for practical purposes. The first task consists

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\*"Prandtl'sche Tragflächen- und Propeller-theorie," Zeitschrift für angewandte Mathematik und Mechanik, 1921, p.206.

\*\*"The Elements of Aerofoil and Airscrew Theory," Cambridge, 1926.

in the evaluation of the lift distribution. From this, we can then determine without difficulty all that is of value regarding the forces and moments acting on the wing.

### III.

We assume a rectangular wing in straight flight. It is understood that all angles of attack will be measured from the position at which the lift disappears, and it is assumed that the lift is a linear function of the angle of attack. The latter statement is practically true, so long as the air flows smoothly along the wing profile, that is, up to the point of separation on the suction side. Our results, therefore, are valid only for angles of attack at which no separation of the flow takes place. The angle of attack of the middle section of the wing (where there is no aileron to alter its shape), is denoted by  $\alpha$ . At the place where the aileron is deflected we shall refer the angle of attack to the original chord. In order to be able to retain the form of presentation of the curves in Figure 10, the lift coefficients, as plotted against the angle of attack, differ from those of the original work. For any given aileron deflection or settings, we consider the altered wing contour as a new wing profile, the aerodynamic characteristics of which we assume to be known. (Experimental results of this kind are found, e.g., in the works of E. A. Landells, W. G. Cowley, and E. A. Griffith, "Tests of New Forms of

Wing Sections with Flaps Extending Over the Whole Length of the Aerofoil." N.P.L. Report 1914/15; and of G. J. Higgins and E. M. Jacobs, "The Effect of a Flap and Ailerons on the N.A.C.A. M-8 Airfoil Section." N.A.C.A. Report No. 260, 1927. The actual aerodynamic angle of attack of the altered profile will be indicated by  $\alpha + \alpha_q$  or  $\alpha - \alpha_q$ . If the altered wing is turned as a whole, with constant aileron setting, from its normal position to the position at which the lift disappears, in both cases we shall consider the angle of attack of the unbroken profile as measured with respect to the chord or some other fixed line of reference. In contrast with the profile of the middle section, the broken profile thus has an aerodynamically effective angle of attack equal to  $\alpha_q$ , which we shall chiefly use in what follows (See Section VI regarding the determination of  $\alpha_q$  from the experimental results). In the matter of opposite aileron settings, we assume that they are equal in both directions and that the air forces produced by the equal angles bear the same relation to the total air forces.

Aileron deflection causes a change in lift distribution. It is evident that the resulting distribution can be considered as made up of two parts, namely, the lift of a wing with constant angle of attack  $\alpha$  over the whole span, and the lift of a wing with an angle of attack  $0^\circ$  at the middle and an angle of attack  $+\alpha_q$  or  $-\alpha_q$  at the ends corresponding to the

length of the ailerons. Since the lift distribution for the first part (constant angle of attack along the whole span) is known, we may confine ourselves to the calculation of the second part. In addition to the most important practical case in which the ailerons are set in opposite directions, the calculations were also made for the case of aileron settings in the same direction. Naturally the length  $l$  of the ailerons (Fig. 1), is the dominating factor for the forces and moments. The calculations were made, therefore, for ailerons of four different lengths. Indicating the span by  $b$ , the lift distribution was determined for the following values of  $2l/b$ :

$$\frac{2l}{b} = 0.234 \quad 0.5 \quad 0.658 \quad 1.0 \quad \text{for opposite settings;}$$

$$\frac{2l}{b} = 0.234 \quad 0.5 \quad 0.658 \quad 1.0 \quad \text{for like settings.}$$

The intermediate aileron lengths can be derived from the curves. The aileron chord does not enter, as such, into the computation, since the altered profile may be regarded as a new wing section with known aerodynamic characteristics. Its effect is expressed by the difference  $\alpha_q$  in the angle of attack. Furthermore, the magnitude of the previously mentioned parameter  $p$  is a measure of the lift distribution. If  $t$  denotes the wing chord, then  $p$  is expressed in the following form:

$$p = \frac{2b}{c_1 t},$$

$c_1$  being defined by the expression  $c_a = 2 c_1 \alpha'$ ,  $c_a$  being



the lift coefficient and  $\alpha'$  being the effective angle of attack expressed in circular measure.  $c_1$  is, therefore, nothing more than a measure of the slope of the lift curve against the angle of attack for infinite span, being identical with  $\frac{1}{2} \left( \frac{d c_a}{d \alpha} \right)_{b=\infty}$  and appropriately taken from experimental data. However, since model experiments are usually conducted with wings of finite span, the value of  $c_1$  for infinite span must be derived therefrom. For the case in which the experiments were conducted with an elliptical wing contour, this conversion becomes simple. According to the wing theory,  $c_1$  acquires the following value:

$$c_1 = \frac{c_a}{2 \alpha - \frac{2 c_a b_0^2}{\pi F_0}} .$$

$F_0$  stands for the wing area,  $b_0$  for the span, and  $\alpha$  for the geometric angle of attack, corresponding to the lift coefficient  $c_a$ , measured from the attitude of zero lift. This formula is not strictly correct, when a rectangular wing is used for the experiments. The error accepted in the compromise amounts, however, to only a few per cent, so that the formula is sufficiently exact for most practical purposes.\* Theoretical researches on the lift of wing surfaces gives for  $c_1$  a value of about  $\pi$ .\*\* Deriving  $c_1$  by the above formula from

\*"Concerning the Exact Calculation of  $c_1$  from the Experimental Results of a Rectangular Wing," H. Glauert's book, p.145 ff.

\*\*Von Mises, "Zur Theorie des Tragflächenauftriebes," Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1917, p.157.

experimental data, one obtains, for the usual profile forms, values between  $2.5$  and  $\pi$ . The original calculations of the lift distribution were made for  $c_1 = \pi$  and for an aspect ratio  $\frac{b}{t} = 2\pi$ , so that the parameter  $p$  took the value of  $4$ . The lift distributions obtained for this case for the given aileron lengths are shown in Figures 2 and 3 for opposite and like aileron settings on both sides of the wing. These curves show only the additional lift produced by the aileron deflection. The total lift distribution is found by adding this lift distribution to that of a wing with constant angle of attack over the whole span corresponding to the angle of attack of the middle wing section, which is already known. The lift distribution for a rectangular wing of aspect ratio  $2\pi$  and constant angle of attack  $\alpha_0$  over the entire span is indicated by dotted lines in both figures.

#### IV.

If we now undertake to determine the lift distribution for other values of the parameter  $p$ , we must, strictly speaking, again find the troublesome solution of the system of eight linear equations, already mentioned. For another value of  $p$  the ordinates of lift distribution given in Figures 2 and 3 do not vary proportionally, but the shape of the whole lift curve varies; or, expressed mathematically, the different parameters  $p$  of corresponding distribution curves cannot be derived from

one another by a common transformation. We presume, however, that in case the value of the parameter for the calculation in question does not differ too much from 4 (which holds true for practical applications), the form of the distribution curve will not differ materially from the calculated curve. For other values of  $p$  the ordinates of the curves would then be easily increased or decreased in a certain ratio, i.e., the distribution curves could be looked upon as related to one another. In order to prove the correctness of this assumption, the exact computation was carried through for the case of codirectional aileron deflections and for  $\frac{2l}{b} = 0.5$ . Furthermore, the exact computation was carried out for a parameter  $p = 5$  and compared with the corresponding results of the case  $p = 4$ . The result of this comparison is shown in Figure 4. Besides the two distribution curves,  $B_4$  and  $B_5$ , of the lift on the semispan for  $p = 4$  and  $p = 5$  (the first being taken from Figure 3), there are also sketched the distributions  $A_4$  and  $A_5$  for constant angle of attack  $\alpha_q$  along the entire span for the same values of  $p$ . The ordinates of the curves  $B_4$  were reduced in the ratio  $\frac{h_5}{h_4}$  of the ordinates of  $A_4$  and  $A_5$ . The distribution  $B_5'$  thus derived is represented by a dotted line. It can be seen that the latter does not quite coincide with the curve  $B_5$ . We will, therefore, consider the distribution  $B_5'$  thus derived from  $B_4$  as equivalent to the actual distribution  $B_5$  and use it in the determination of the air

forces and moments. The resulting error is negligible for most practical applications. Since the ordinate ratio  $\frac{h_5}{h_4}$  is somewhat different from place to place (the curves  $A_4$  and  $A_5$  themselves being uncorrelated), in the computation we shall at times use the ratio in the neighborhood of the maximum  $B_4$  as indicated in Figure 4.

## V.

From the lift distribution thus found, the additional forces and moments acting on the wing can now be determined. In the most important practical cases in which the aileron settings are in opposite directions, the following forces and moments act upon the wing:

1. A rolling moment  $M_x$  about the longitudinal axis,
2. An additional induced drag  $W_1$ ,
3. A yawing moment  $M_z$  about the vertical axis.

The total lift in this case remains unchanged. The yawing moment  $M_z$  is due to the fact that the unequal lift distribution over the two wings produce, a different induced drag on each wing. The additional induced drag is caused by the considerable deviation from the most favorable lift distribution, which is known to be elliptical.

Due to the complete symmetry, no yawing moment occurs for like settings of the ailerons. Pitching moments fall outside the field of our observations. Here we have:

4. An additional lift  $A_1$ ,
5. An additional induced drag  $W_2$ .

As shown in the detailed study, these quantities may be expressed in the following manner:

For opposite aileron settings:

$$\text{Rolling moment} \quad M_X = q b^3 \xi \alpha_q$$

$$\text{Additional induced drag} \quad W_1 = q b^2 \eta \alpha_q$$

$$\text{Yawing moment} \quad M_Z = q b^3 \xi \alpha_q$$

For like aileron settings:

$$\text{Additional lift} \quad A_1 = q b^2 \lambda \alpha_q$$

$$\text{Additional induced drag} \quad W_2 = q b^2 \kappa \alpha_q$$

In these equations, besides the well-known quantities ( $q$  = dynamic pressure), there appear also the nondimensional coefficients  $\xi$ ,  $\eta$ ,  $\xi$ ,  $\lambda$ , and  $\kappa$ . These coefficients depend on the form of the lift distribution and are the result of integrations over the span. The approximate values of these coefficients, deduced from the calculations for  $p = 4$  with the help of the process given in Section IV, are reproduced in the curve groups of Figures 5 to 9. They are dependent on the ratio  $\frac{2l}{b}$  for different values of the parameter  $p$ ; namely, for  $p = 4, 4.5, 5.0, 5.5$ , and  $6$ . The coefficients  $\xi, \dots, \kappa$  for all aileron lengths and for all values of  $p$  occurring in practice may be interpolated from these diagrams, and the desired moments and forces may be obtained in a simple manner by

substitution in the above formulas. The values of these coefficients are also given in the following tables.

1. Coefficients  $\xi$  of the Rolling Moment  $M_x$

$\frac{2l}{b} =$	0.234	0.500	0.658	1.000
p = 4.0	0.047	0.100	0.114	0.135
4.5	0.044	0.092	0.105	0.125
5.0	0.041	0.085	0.096	0.114
5.5	0.039	0.079	0.089	0.105
6.0	0.038	0.074	0.084	0.098

2. Coefficients  $\eta$  of the Additional Induced Drag  $W_1$

$\frac{2l}{b} =$	0.234	0.500	0.658	1.000
p = 4.0	0.0462	0.1114	0.144	0.198
4.5	0.0402	0.0939	0.121	0.165
5.0	0.0353	0.0806	0.103	0.140
5.5	0.0318	0.0696	0.0883	0.120
6.0	0.0297	0.0607	0.0770	0.104

3. Coefficients  $\xi$  of the Yawing Moment  $M_z$

$\frac{2l}{b} =$	0.234	0.500	0.658	1.000
p = 4.0	0.0492	0.0895	0.0965	0.109
4.5	0.0428	0.0755	0.0812	0.0910
5.0	0.0376	0.0648	0.0690	0.0772
5.5	0.0339	0.0559	0.0592	0.0659
6.0	0.0316	0.0488	0.0520	0.0570

4. Coefficients  $\lambda$  of the Additional Lift  $A_1$ 

$\frac{2l}{b} =$	0.234	0.500	0.658	0.826	1.000
p = 4.0	0.130	0.326	0.442	0.587	0.729
4.5	0.121	0.299	0.405	0.536	0.663
5.0	0.113	0.277	0.374	0.494	0.607
5.5	0.106	0.258	0.346	0.456	0.559
6.0	0.100	0.241	0.324	0.425	0.519

5. Coefficients  $\kappa$  of the Additional Induced Drag  $W_2$ 

$\frac{2l}{b} =$	0.234	0.500	0.658	0.826	1.000
p = 4.0	0.056	0.116	0.137	0.163	0.178
4.5	0.048	0.098	0.115	0.136	0.147
5.0	0.042	0.083	0.098	0.115	0.123
5.5	0.037	0.073	0.084	0.098	0.105
6.0	0.033	0.063	0.074	0.085	0.090

## VI.

The quantities  $c_l$  and  $\alpha_q$  for the determination of the forces and moments for a given case will be taken from profile tests with full-length ailerons or flaps. The data obtained by Higgins and Jacobs in the high-pressure tunnel, are here reproduced by way of illustration. The lift coefficients  $c_a$  for different aileron settings  $\beta$ , as well as the airfoil used (the N.A.C.A. M-6 with an aspect ratio of 6) are shown in Figure 10. The aileron here extends along the whole length of the wing model, and the aileron chord is 20 per cent of the wing chord. It can be seen from the diagram that the lift curves run almost parallel to one another, that is, that the value

$c_1 = \frac{1}{2} \frac{d c_a}{d \alpha}$  is very nearly constant for various aileron settings. From these curves we learn, furthermore, that the absolute value of the lift variation for positive and negative aileron settings, for the profile in question, is very nearly the same, so that the assumption made in this connection under Section III holds good. However, it should be noted that a symmetrical aileron effect cannot be expected for all profiles (i.e., equal lift variation on both sides for equal aileron settings in opposite directions), and that occasionally  $c_1$  and  $\alpha_q$  may have different values for the two wings. In cases in which the asymmetry is not pronounced, fairly good results are obtained by determining the air forces and moments for the two wings separately and simply adding the values thus obtained, or (which amounts to the same thing) by taking the arithmetical mean of the quantities  $c_1$  and  $\alpha_q$  or of  $c_1'$  and  $\alpha_q'$  of the values found for both wings together.

If, by way of example, we assume that the N.A.C.A. profile M-6 has an angle of attack of  $3^\circ$  to the direction of flight, as shown by the dotted reference line, the angle of attack measured from the attitude of zero lift would be about  $\alpha = 4.5^\circ$ . If we further assume that the ailerons are deflected by  $\beta = \pm 10^\circ$ , we then see from the diagram that, for the angles of attack within the aileron working range,  $\alpha + \alpha_q = +9.2^\circ$  and  $\alpha - \alpha_q = -0.2^\circ$ . Hence the angle  $\alpha_q$  is  $4.7^\circ$ . From Figure 10 we also learn that, for the customary form of lift-curve rep-



resentation (i.e., even though, for the deflected aileron, the angle of attack is computed from the original reference line), the angle  $\alpha_q$  is only the angular difference between the intersections on the angle-of-attack axis of the curves  $\beta = 0^\circ$  and  $\beta = 10^\circ$ .

### S u m m a r y

In an earlier work the lift distribution, produced by aileron deflections (in the same and in opposite directions) for a rectangular wing in straight flight, was determined on theoretical principles for a certain case, and the air forces and moments produced by the ailerons were derived therefrom. In order to widen the field of application, approximate values are here deduced from the former results, with the help of which the acting forces and moments may be calculated for all cases occurring in practice. The results are expressed by non-dimensional coefficients, which are presented in the form of diagrams. For any given case the corresponding coefficients are derived from these diagrams (by interpolation, when necessary), and, by inserting them in the given formulas, the desired forces and moments can be easily ascertained.

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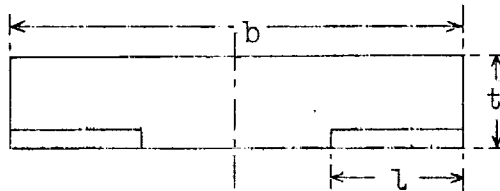


Fig. 1

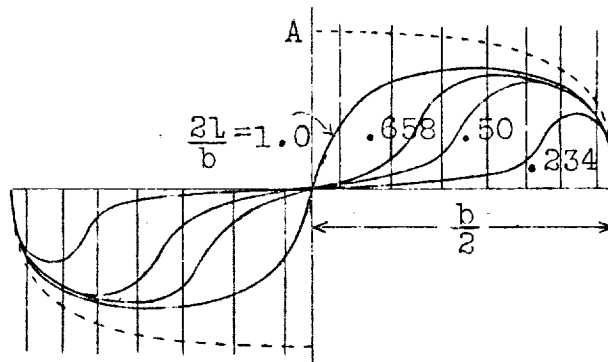


Fig. 2 Distribution of additional lift for different aileron lengths and opposite aileron settings.

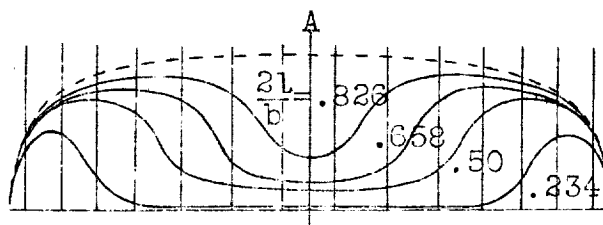


Fig. 3 Distribution of additional lift for different aileron lengths and like aileron settings.

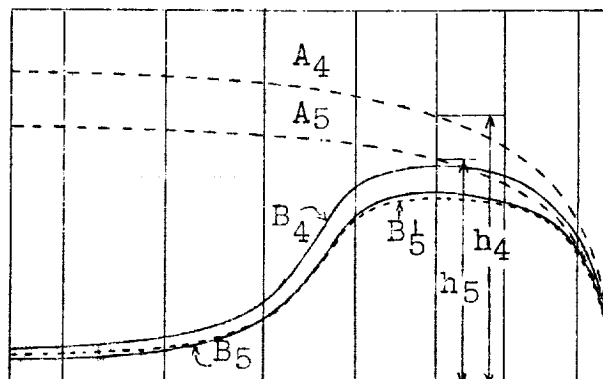


Fig. 4.

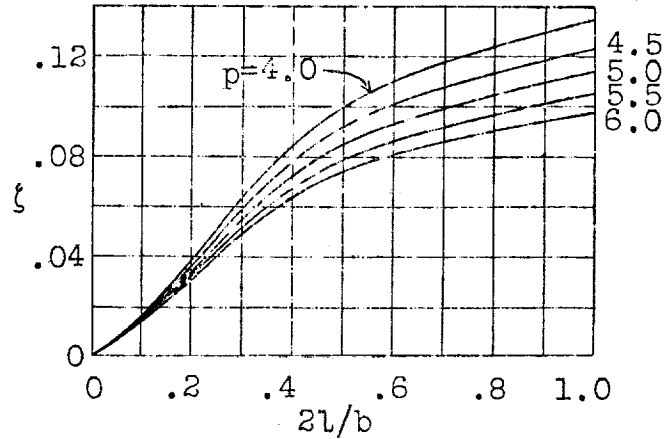


Fig. 5 Coefficient  $\xi$  of rolling moment.

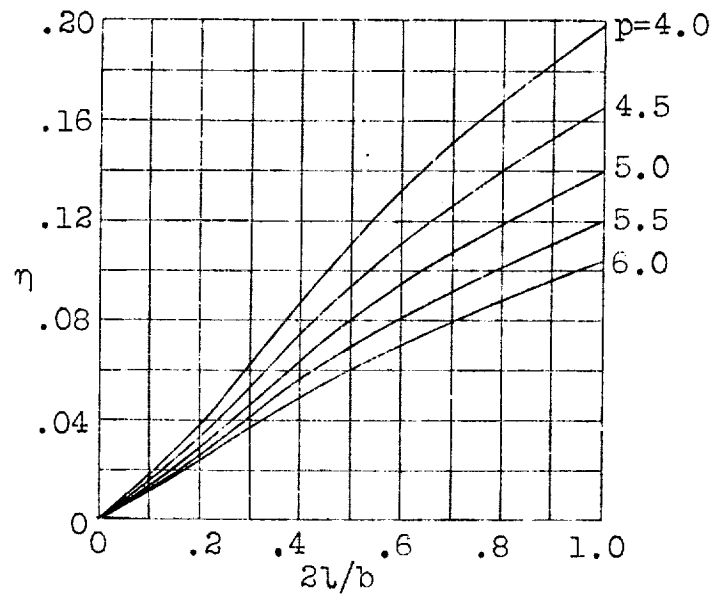


Fig. 6 Coefficient  $\eta$  of additional induced drag for opposite aileron settings.

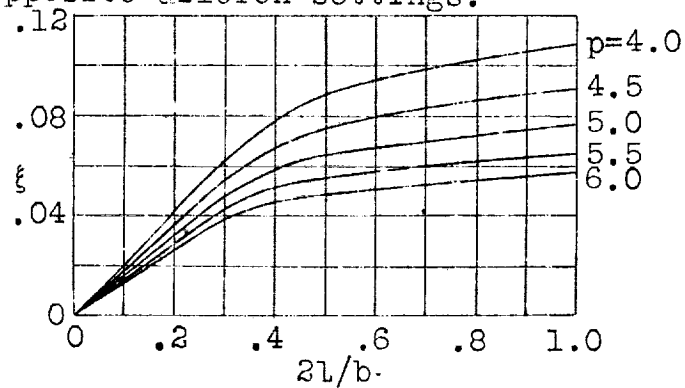


Fig. 7 Coefficients  $\xi$  of yawing moment.

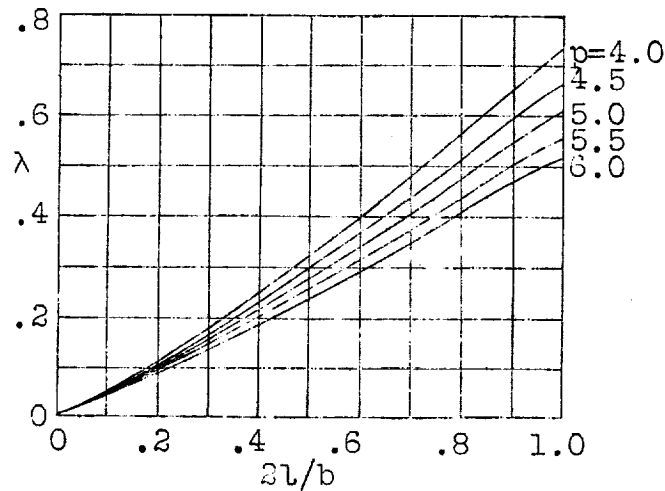


Fig.8 Coefficients  $\lambda$  of additional lift for like aileron settings.

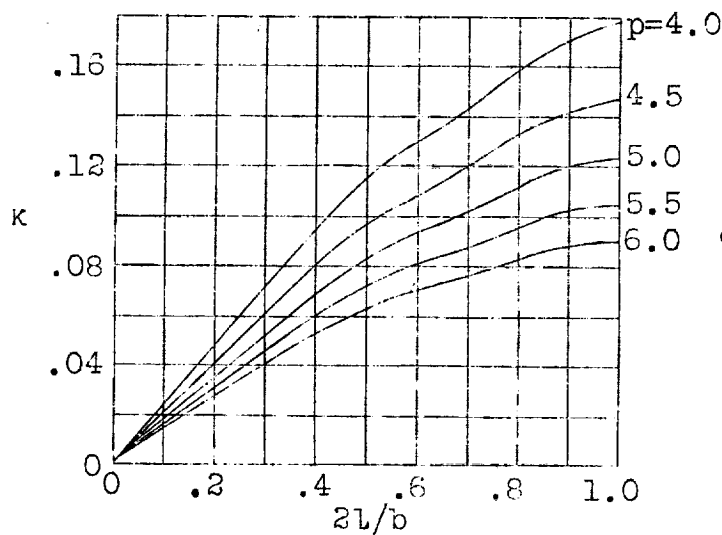


Fig.9

Coefficient  $\kappa$  of additional induced drag for like aileron settings.

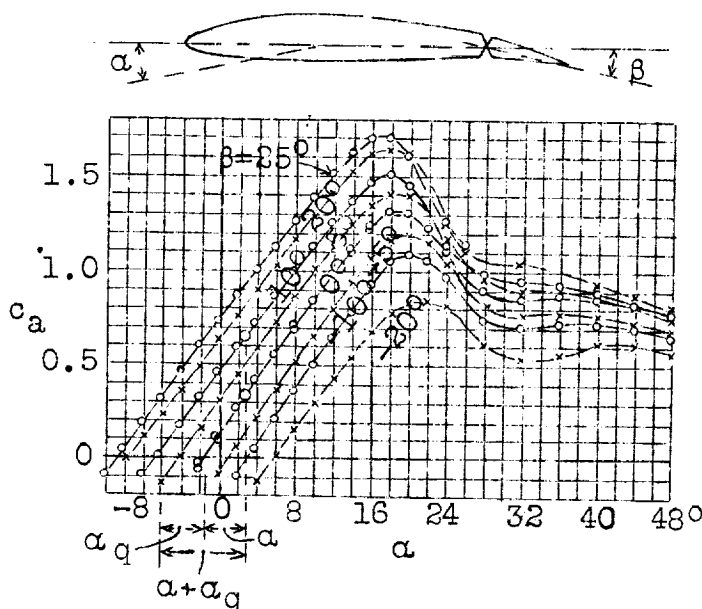


Fig.10

Lift coefficient  $c_a$  vs. angle of attack for different settings,  $\beta$  of full-length ailerons, on N.A.C.A.-M6 airfoil.